CMOS ANALOG INTEGRATED FILTERS

2001. 7. 5.

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OUTLINE

• Introduction to Analog Integrated Filters

• Introduction to Transconductance (Gm)-C Filters
  ✓ Transconductance Circuits
  ✓ High-Order Gm-C Filters
  ✓ Design Issues for Gm-C Filters
  ✓ Design Examples

• Introduction to Switched-Capacitor (SC) Filters
  ✓ Switched-Capacitor Integrators
  ✓ High-Order SC Filters
  ✓ Design Issues for SC Filters
  ✓ Design Examples

• Simulation Exercises
Introduction
Mixed-Signal Systems

Performance Requirements

AFE1 < AFE2
Analog Integrated Filters

- Analog Filters vs Digital Filters
  - :-) Speed, Power Dissipation, Silicon Area
  - :-( Dynamic Range, Programmability, Controllability

- Continuous-Time Filters
  - Gm-C Filter, MOSFET-C Filter, Active R-C Filter, ...
  - No Pre- & Post- Processing Required
  - Tuning Circuits Required

- Discrete-Time Filters
  - Switched-Capacitor (SC) Filter, Switched-Current Filter
  - Pre- & Post- Processing Required
  - Desirably Accurate
Filter Frequency Response

- **Magnitude Characteristics**

- **Phase Characteristics**

- **Operating Types**
  - Lowpass, Highpass, Bandpass, Bandreject, Allpass
  - Frequency Transformation from LPF Prototype

- **Response Types**
  - Butterworth, Chebyshev (I,II), Elliptic, Equiripple
**High-Order Filter Implementation 1**

**Cascading of Biquad Blocks**

\[
H(z) = H_1(z)...H_i(z)...H_{N/2}(z)
\]

\[
H(z) = \prod_{i=1}^{N/2} H_i(z) = \prod_{i=1}^{N/2} \frac{b_2 z^2 + b_1 z + b_0}{a_2 z^2 + a_1 z + a_0}
\]

- Easy Adjustment of Frequency Characteristics
- Pole/Zero Pair Matching
- Biquad Permutation
- Gain Distribution
High-Order Filter Implementation 2

• Implemented with Many Integrators
• Less Sensitive to Variations of Components
• Easy to Obtain
• Frequency Transformation
• Impedance Transformation

RLC Prototype

\[ I_0 = \frac{(X - V_1)}{R_S} \]
\[ V_1 = \frac{(I_0 - I_2)}{sC_1} \]
\[ I_2 = \frac{(V_1 - V_3)}{sL_2} \]
\[ V_3 = \frac{(I_2 - I_4)}{sC_3} \]
\[ I_4 = \frac{(V_3 - I_4R_L)}{sL_2} \]
Continuous-Time Filter Response

\[
H(s) \equiv \frac{X(s)}{Y(s)} = \frac{b_1 s^M + b_2 s^{M-1} + \cdots + b_M s + b_{M+1}}{a_1 s^N + a_2 s^{N-1} + \cdots + a_N s + a_{N+1}}
\]
Discrete-Time Filter Response

\[ H(z) = \frac{N(z)}{D(z)} = \frac{b_1 z^M + b_2 z^{M-1} + \cdots + b_M z + b_{M+1}}{a_1 z^N + a_2 z^{N-1} + \cdots + a_M z + a_{M+1}} \]
Continuous-Time Filter
(Gm-C Filter)
**Gm Amplifier Basics**

- **Transconductance (Gm) Amplifier**
  
  - Single-Ended
  
  
  ![Single-Ended Gm Amplifier](image1.png)

  ![Fully-Differential Gm Amplifier](image2.png)

- **Transconductance Amplifier Characteristics**
  
  - Linearity: \( i_{out} \equiv G_m \cdot v_{in} = G_m \cdot (v_{in}^+ - v_{in}^-) \)
  
  - I/O Impedances: \( R_{in} = \infty, \quad R_{out} = \infty \)
  
  - Operating (Dynamic) Range
  
  - Frequency Characteristics
  
  - Electrically Programmable Gm Value
**Gm Amp Circuit Examples**

- **Simple Differential Pair Example**

  ![Diagram of a simple differential pair example](image)

  \[
  i_1 = \frac{\beta}{2} (v_{in}^+ - v_x - V_{TH})^2 \\
  i_2 = \frac{\beta}{2} (v_{in}^- - v_x - V_{TH})^2
  \]

  \[
  i_{out} \equiv i_1 - i_2 = \frac{\beta}{2} v_{in} \sqrt{\frac{2I_{SS}}{\beta/2} - v_{in}^2}
  \]

  For large \( I_{SS} \), small \( \beta \) & \( v_{in} \)

  \[
  i_{out} \approx \sqrt{\beta I_{SS}} \cdot v_{in}
  \]

  \[
  \therefore G_m \approx \sqrt{\beta I_{SS}}
  \]
**Differential Pair w/ Degenerate Resistor Example**

\[ i_1 = i_{R_C} + I_B \]
\[ i_2 + i_{R_C} = I_B \]
\[ i_1 - i_2 = 2 \cdot i_{R_C} \]

\[ v_+ - v_{gs_1} - i_{R_C} R_C + v_{gs_2} = v_- \]
\[ i_{out} \equiv i_1 - i_2 = \frac{2}{R_C} \{ \Delta v_{in} - (v_{gs_1} - v_{gs_2}) \} \]

for large \( \beta \) and small \( \sqrt{i_1} - \sqrt{i_2} \)

\[ v_{gs_1} - v_{gs_2} = (v_{TH} + \sqrt{2i_1/\beta}) - (v_{TH} + \sqrt{2i_2/\beta}) = \sqrt{2/\beta} (\sqrt{i_1} - \sqrt{i_2}) \approx 0 \]

\[ \therefore G_m \equiv \frac{\partial i_{out}}{\partial \Delta v_{in}} = \frac{i_{out}}{\Delta v_{in}} \equiv \frac{2}{R_C} \]
**Differential Pair w/ Degenerate Resistor & Feedback Example**

Negative feedback loops ($M_1$-$M_5$-$M_3$ & $M_2$-$M_6$-$M_4$) forces constant currents $I_{B1}$ to flow through $M_1$ and $M_2 \Rightarrow v_{gs1} = v_{gs2}$
**TC Amp using MOS in Saturation**

- **Current in Saturation MOSFET**
  \[ i_{ds} = \frac{\beta}{2} (v_{gs} - V_{TH})^2 \]

- **Basic Principle**
  \[ A^2 - B^2 = (A + B) \cdot (A - B) = k \cdot (v^+_\text{in} - v^-_{\text{in}}) \]

- **Composite Transistor**
  \[ I_D = \frac{\beta_{eq}}{2} (V_{GSeq} - V_{THeq})^2 \]
  \[ V_{GSeq} \equiv V_{GSN} + V_{SGP} \]
  \[ V_{THeq} \equiv V_{THN} + |V_{THP}| \]
  \[ \beta_{eq} \equiv \frac{1}{2} \left( \frac{\beta_N \beta_P}{\sqrt{\beta_N/2} + \sqrt{\beta_P/2}} \right)^2 \]
  \[ i_{out} = 2\beta_{eq} \left[ V_{FV} - V_{THeq} \right] \cdot (v_1 - v_2) \]
**TC Amp using MOS in Linear**

- **Current in Saturation MOSFET**
  \[ i_{ds} = \beta \left( (v_{gs} - V_{TH})v_{ds} - \frac{1}{2}v_{ds}^2 \right) \]

- **Basic Principle**
  \( (A - C^2) - (B - C^2) = A - B \propto v_{in}^+ - v_{in}^- \)

- **Example**
  \[ i_{out} = i_p - i_m = (i_1 + i_3) - (i_2 + i_4) = (i_1 - i_4) - (i_2 - i_3) \]

  \[ i_1 - i_4 = 2i_B \]

  \[ = 2\beta \left[ (V_B - V_{B2} - V_{TH})(V_{B1} - V_{B2}) - \frac{1}{2}(V_{B1} - V_{B2})^2 \right] \]

  \[ i_2 - i_3 = 2i_A \]

  \[ = 2\beta \left[ (V_A - V_{A2} - V_{TH})(V_{A1} - V_{A2}) - \frac{1}{2}(V_{A1} - V_{A2})^2 \right] \]
**Gm-C Integrator**

The Gm-C integrator is a fundamental component in analog circuits, characterized by its transfer function, which relates the output voltage \( v_{out} \) to the input voltage \( v_{in} \) and current, with the capacitor \( C \).

\[
H(s) \equiv \frac{v_{out}}{v_{in}} = \frac{G_m}{sC}
\]

where \( G_m \) is the transconductance gain.

- **Critical Parameters**
  - **Unity Gain Frequency** \( \omega_{unity} \)
    \[
    \omega_{unity} = \frac{G_m}{C}
    \]
  - **Phase Margin** \(-90^\circ\) and **Gain Margin** \(-6dB/dec\)

These parameters are crucial for determining the filter frequency characteristics of the integrator.
**1\textsuperscript{st}-Order Gm-C Filter**

\[ H(s) \equiv \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{C_x}{C_A + C_x} s + \frac{G_{m1}}{C_A + C_x} \left( 1 + \frac{G_{m2}}{s(\omega_0)} \right) \]

\[ H(s) \equiv \frac{k_1 s + k_0}{s + \omega_0} \]

\[ G_{m1} = k_0 \cdot (C_A + C_x) \]

\[ G_{m2} = \omega_0 \cdot (C_A + C_x) \]

\[ C_x = \frac{k_1}{1 - k_1} C_A \]
2nd-Order Gm-C Filter (Biquad)

\[
H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{C_X}{C_B + C_X} s^2 + \frac{G_{m5}}{C_B + C_X} s + \frac{G_{m2}G_{m4}}{C_A(C_B + C_X)} \longleftrightarrow k_2 s^2 + k_1 s + k_0 = \frac{s^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}
\]
**Characteristic Function**

\[ r = s^2 + \left( \frac{\omega_0}{Q} \right)s + \omega_0^2 \]

\[ \omega_0 = \frac{G_{m1}G_{m2}}{\sqrt{C_A \left( C_B + C_X \right)}} \]

\[ Q = \frac{G_{m1}G_{m2}}{\sqrt{G_{m3}^2 \frac{C_B + C_X}{C_A}}} \]

**Filtering Operations**

\[ k_2 = \frac{C_X}{C_B + C_X} \quad k_1 = \frac{G_{m5}}{C_B + C_X} \quad k_0 = \frac{G_{m2}G_{m4}}{C_A \left( C_B + C_X \right)} \]

- Lowpass: \( k_2 = k_1 = 0 \)
- Highpass: \( k_1 = k_0 = 0 \)
- Bandpass: \( k_2 = k_0 = 0 \)
6th-order Elliptic BPF

- Passband: 480kHz~520kHz
- Passband Ripple: < 0.05dB
- Stopband Attenuation: > 80dB

- $H_1(s)$
  - $G_{m1} = G_{m2} = 17.06\mu$, $G_{m3} = 0.80\mu$
  - $G_{m4} = 4.18\mu$, $G_{m5} = 0\mu$
  - $C_A = C_B = 5p$, $C_X = 0.319p$

- $H_2(s)$
  - $G_{m1} = G_{m2} = 16.19\mu$, $G_{m3} = 1.52\mu$
  - $G_{m4} = 0.21\mu$, $G_{m5} = 0\mu$
  - $C_A = C_B = 5p$, $C_X = 0.319p$

- $H_3(s)$
  - $G_{m1} = G_{m2} = 14.89\mu$, $G_{m3} = 0.67\mu$
  - $G_{m4} = 0\mu$, $G_{m5} = 1.89\mu$
  - $C_A = C_B = 5p$, $C_X = 0p$
**High-Order Gm-C Filter from RLC**

\[ I_0 = \frac{(X - V_1)}{R_s} \]
\[ V_1 = \frac{(I_0 - I_2)}{sC_1} \]
\[ I_2 = \frac{(V_1 - V_3)}{sL_2} \]
\[ V_3 = \frac{(I_2 - I_4)}{sC_3} \]
\[ I_4 = \frac{(V_3 - I_4 R_L)}{sL_2} \]

\[ V_0 = A_s \left( X - V_1 \right) \]
\[ V_1 = \frac{(V_0 - V_2)}{s\tau_1} \]
\[ V_2 = \frac{(V_1 - V_3)}{s\tau_2} \]
\[ V_3 = \frac{(V_2 - V_4)}{s\tau_3} \]
\[ V_4 = \frac{(V_3 - Y)}{s\tau_4} \]

Frequency Transformation & Impedance Transformation Required
**Non-Idealities**

- **Finite Output Resistance**
  \[ H(s) = \frac{G_m R_{out}}{1 + sR_{out} C} \]
  
  - Finite DC Gain: BPF, HPF
  - Reduced Phase (> -90°): Q decreases
  - Cascode Output Stage & Compensation Technique

- **Finite Parasitic Pole Frequency**
  \[ H(s) = \frac{G_m R_{out}}{1 + sR_{out} C} \cdot \frac{1}{1 + s/p_{int\_nual}} \]
  
  - Increased Phase (< -90°): Q increases
  - Careful Design

- **Input Capacitance Loading**
  \[ H(s) = \frac{G_m}{s(C + C_{input})} \]
  
  - Frequency Change
  - Dummy Cells
**Linearity Problems**

- **Nonlinearity**
  \[ I_{\text{out}}(t) = a_1 V_{\text{in}}(t) + a_2 [V_{\text{in}}(t)]^2 + a_3 [V_{\text{in}}(t)]^3 + \cdots \]

- **Harmonic Distortion**
  \[ V_{\text{in}}(t) = A \sin(\omega_0 t) \]
  \[ I_{\text{out}}(t) = I_1 \sin(\omega_0 t) + I_2 \sin(2\omega_0 t) + I_3 \sin(3\omega_0 t) + \cdots \]
  \[ \text{THD (dB)} = 10 \log \left( \frac{I_2^2 + I_3^2 + I_4^2 + \cdots}{I_1^2} \right) \]

- **Inter Modulation Distortion**
  \[ V_{\text{in}}(t) = A [\sin(\omega_1 t) + \sin(\omega_2 t)] \]
  \[ \omega_1 \approx \omega_2 \]
  - **IMD2** \( \omega_1 + \omega_2, \ \omega_1 - \omega_2 \)
  - **IMD3** \( \omega_1 + 2\omega_2, \ 2\omega_1 + \omega_2, \ 2\omega_1 - \omega_2, \ \omega_1 - 2\omega_2 \)
**Dynamic Range Performance**

- **IIP3**: Input-Referred Third-Order Intercept Point
- **IP_{1dB}**: Input-Referred 1-dB Compression Point
- **SFDR**: Spurious-Free Dynamic Range
Gm Variations

- Values of $G_m$ might change due to process variation, temperature, aging, ...

- **Trimming** after chip fabrication possible, but one-time event.

- freq. change
- Q change
On-Chip Tuning Example for Biquad

$$H(s) = \frac{D(s)}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$
Recent Advance Example 1

55 Mbps Equalizer for Magnetic Storage Read Channels

- 4-pole & 1-zero
- Equalization
- for Peak-Detect Scheme
- 2.0-μm CMOS Technology
- SNR > 57 dB, THD < 1%
- Power = 40 mW
- UCLA (IEEE JSSC, 1994)
Recent Advance Example 2

7-th Order Equiripple Gm-C Filter

• 0.4-µm CMOS Technology
• \( f_{\text{cutoff}} = 50 \) MHz
• THD < 1 %, Power = 100 mW (3 V)
• Hitachi (IEEE ISSCC, 1997)
Recent Advance Example 3

Raised-Cosine Matched Filter for 480 MHz QPSK Demodulator for DBS

Analog Devices (IEEE ISSCC, 1997)
Recent Advance Example 4

16th-order BPF for J-PDC

- 2 x 8th-order BPF
- Average of $f_0$ : 450.5kHz
- $\sigma$ of $f_0$ : ± 1.5kHz
- Tuning Range of $f_0$ : 240~770kHz
- 0.35-µm CMOS
- Power : 4.8mA @ 2.5V
- Active Area : 2.5mm$^2$
- Fujitsu

(IEEE ISSCC, 1999)
Recent Advance Example 5

7th-order Equiripple Group Delay LPF

- $1 + 3 \times \text{Biquads}$
- $f_{3dB} : 450.5 \text{kHz}$
- $f_{3dB}$ Accuracy: $\pm 10\%$
- Group Delay Acc. : $\pm 5\%$
- 0.25-$\mu$m CMOS
- Power Supply : 2.5V
- Chip Area : 3mm$^2$
- Texas Instruments

(IEEE ISSCC, 1999)