CMOS ANANLOG INTEGRATED FILTERS

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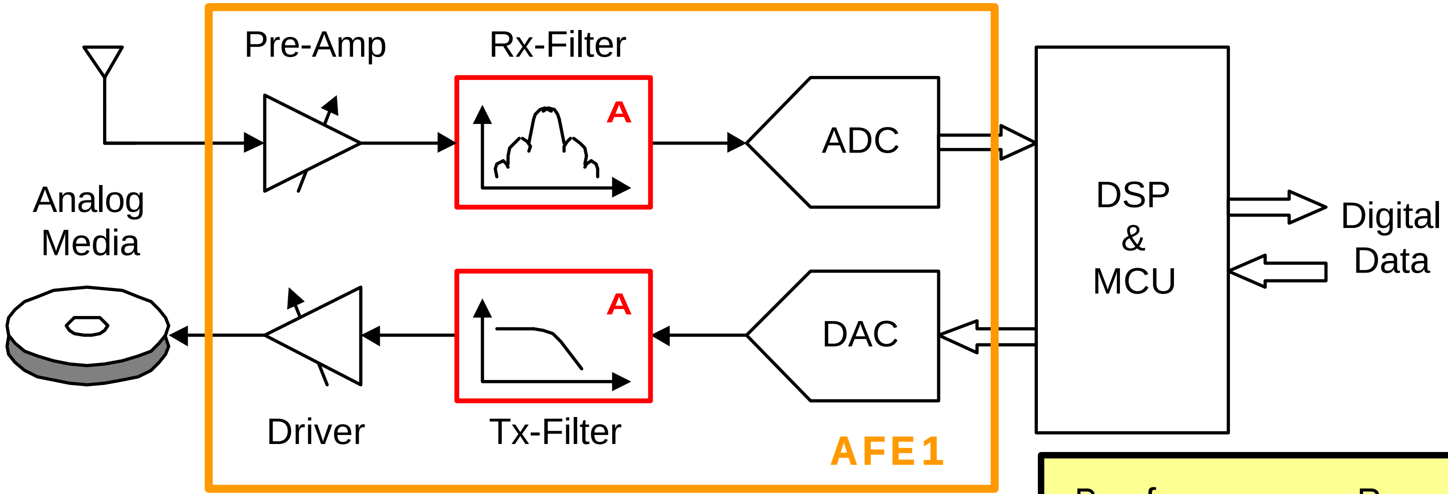
OUTLINE

- **Introduction to Analog Integrated Filters**
- **Introduction to Transconductance (Gm)-C Filters**
  - ✓ Transconductance Circuits
  - ✓ High-Order Gm-C Filters
  - ✓ Design Issues for Gm-C Filters
  - ✓ Design Examples
- **Introduction to Switched-Capacitor (SC) Filters**
  - ✓ Switched-Capacitor Integrators
  - ✓ High-Order SC Filters
  - ✓ Design Issues for SC Filters
  - ✓ Design Examples
- **Simulation Exercises**

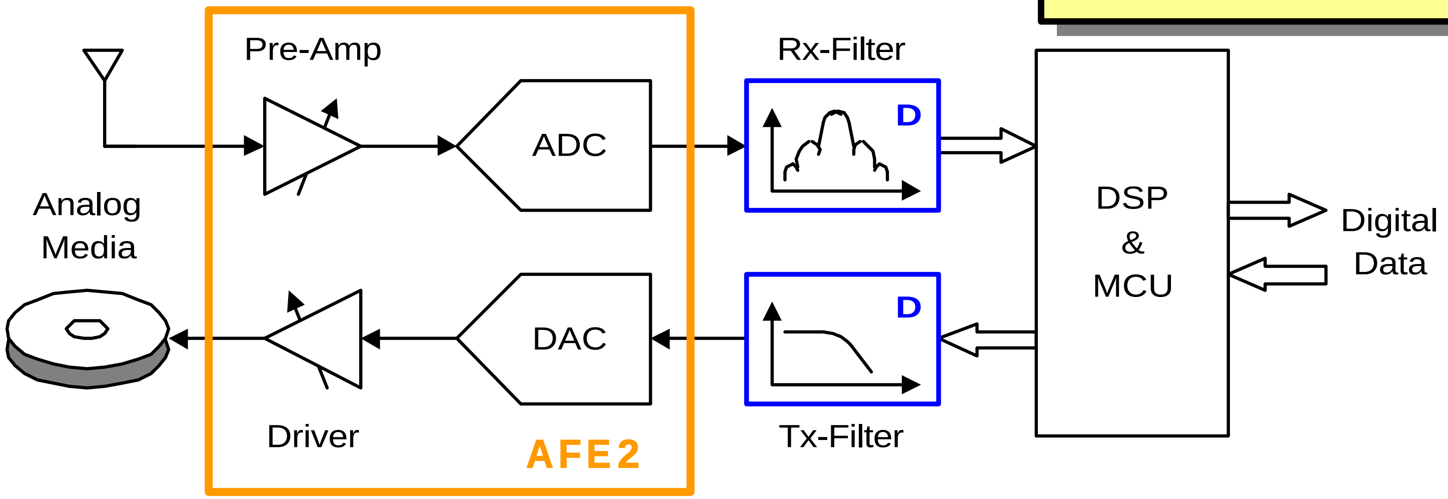
Introduction

Mixed-Signal Systems



Performance Requirements

AFE1 < AFE2



Analog Integrated Filters

- **Analog Filters vs Digital Filters**

  :-) Speed, Power Dissipation, Silicon Area

  :-(Dynamic Range, Programmability, Controllability

- **Continuous-Time Filters**

  → Gm-C Filter, MOSFET-C Filter, Active R-C Filter, …

  → No Pre- & Post- Processing Required

  → Tuning Circuits Required

- **Discrete-Time Filters**

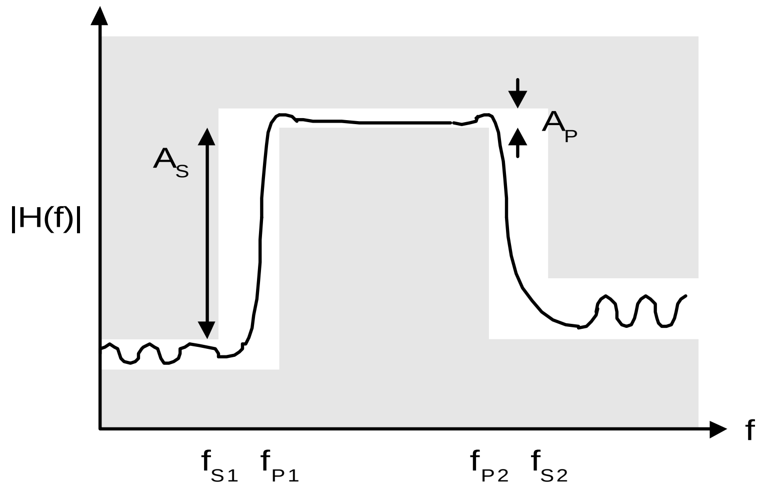
  → Switched-Capacitor (SC) Filter, Switched-Current Filter

  → Pre- & Post- Processing Required

  → Desirably Accurate

Filter Frequency Response

- **Magnitude Characteristics**



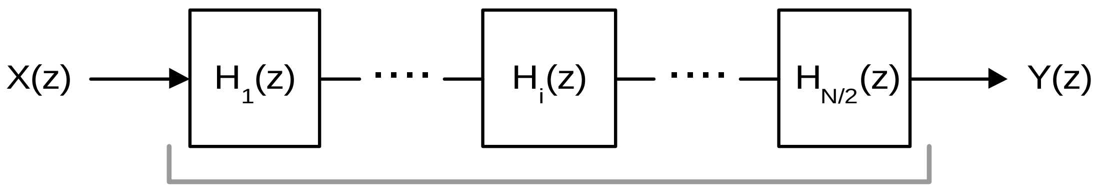
- **Phase Characteristics**

- **Operating Types**
  - ➔ Lowpass, Highpass, Bandpass, Bandreject, Allpass
  - ➔ Frequency Transformation from LPF Prototype

- **Response Types**
  - ➔ Butterworth, Chebyshev (I,II), Elliptic, Equiripple

High-Order Filter Implementation 1

Cascading of Biquad Blocks



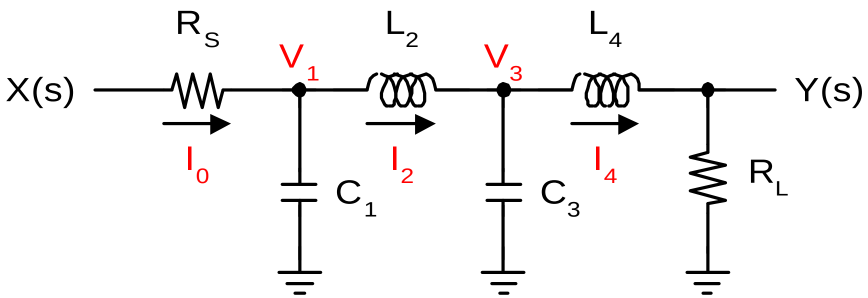
$$H(z) = H_1(z)\ldots H_i(z)\ldots H_{N/2}(z)$$

$$H(z) = \prod_{i=1}^{N/2} H_i(z) = \prod_{i=1}^{N/2} \frac{b_2 z^2 + b_1 z + b_0}{a_2 z^2 + a_1 z + a_0}$$

- **Easy Adjustment of Frequency Characteristics**
- **Pole/Zero Pair Matching**
- **Biquad Permutation**
- **Gain Distribution**

High-Order Filter Implementation 2

RLC Prototype

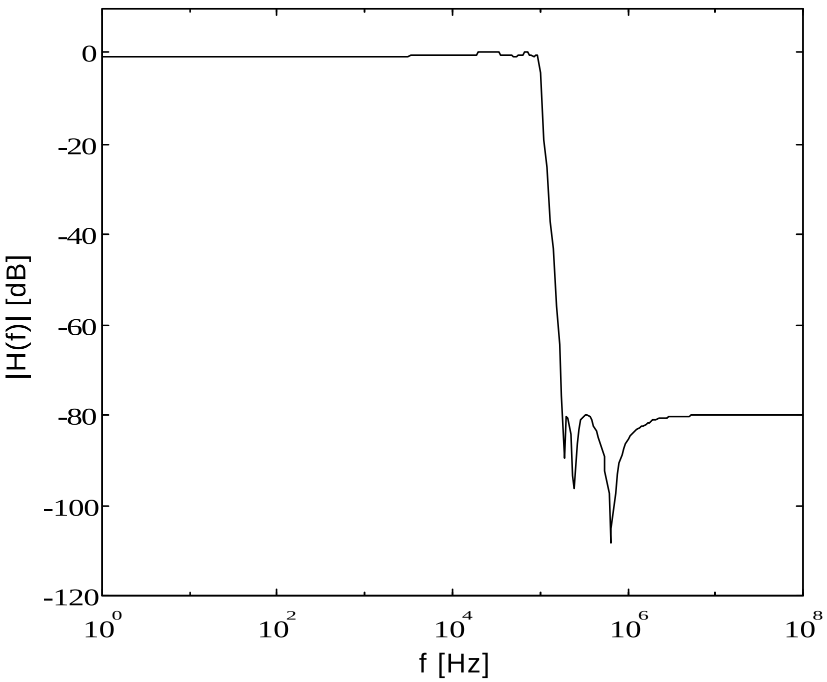


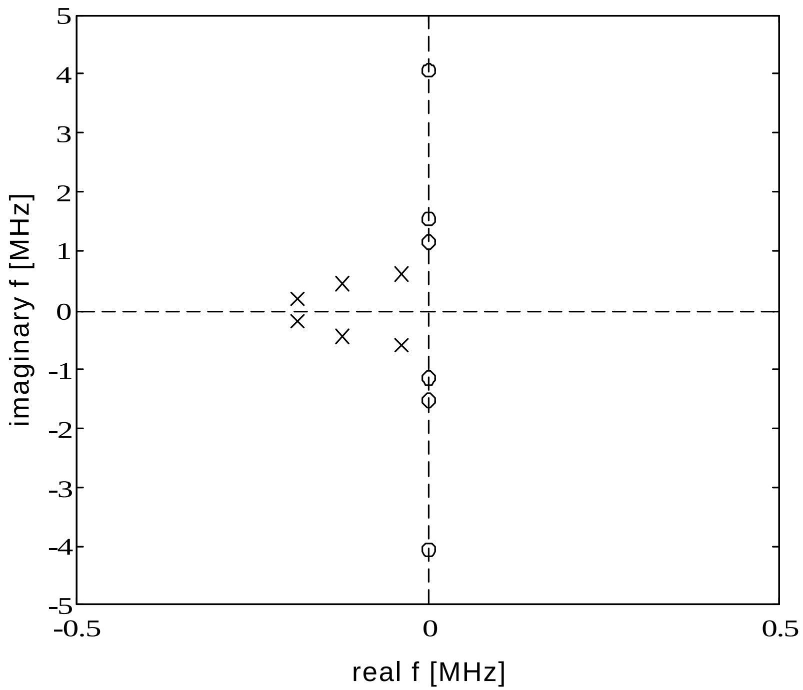
$$I_0 = (X - V_1)/R_S$$
$$V_1 = (I_0 - I_2)/sC_1$$
$$I_2 = (V_1 - V_3)/sL_2$$
$$V_3 = (I_2 - I_4)/sC_3$$
$$I_4 = (V_3 - I_4 R_L)/sL_2$$

- **Implemented with Many Integrators**
- **Less Sensitive to Variations of Components**
- **Easy to Obtain**
- **Frequency Transformation**
- **Impedance Transformation**

Continuous-Time Filter Response

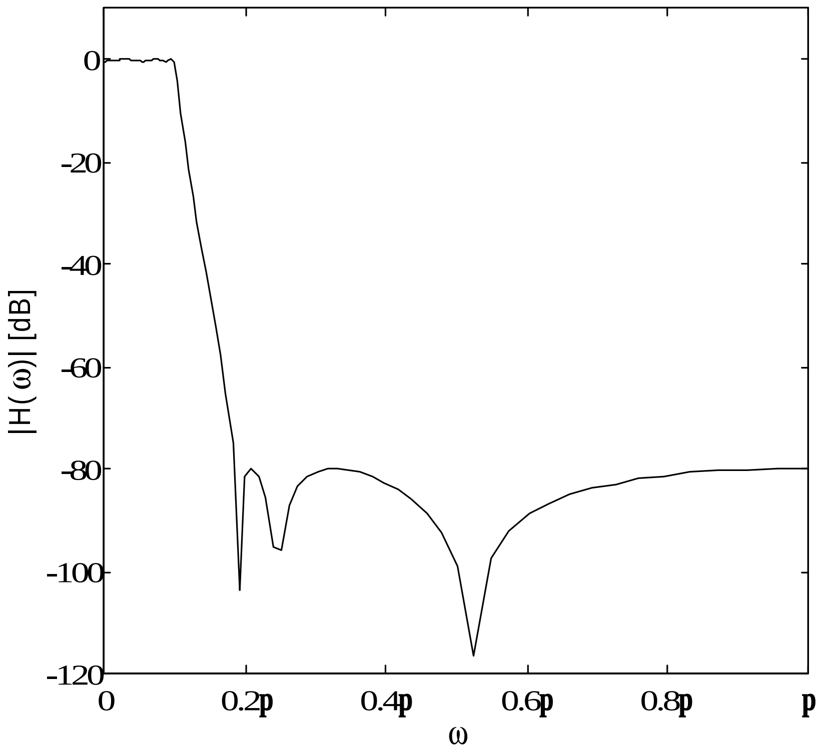
$$H(s) \equiv \frac{X(s)}{Y(s)} = \frac{b_1 s^M + b_2 s^{M-1} + \cdots + b_M s + b_{M+1}}{a_1 s^N + a_2 s^{N-1} + \cdots + a_N s + a_{N+1}}$$

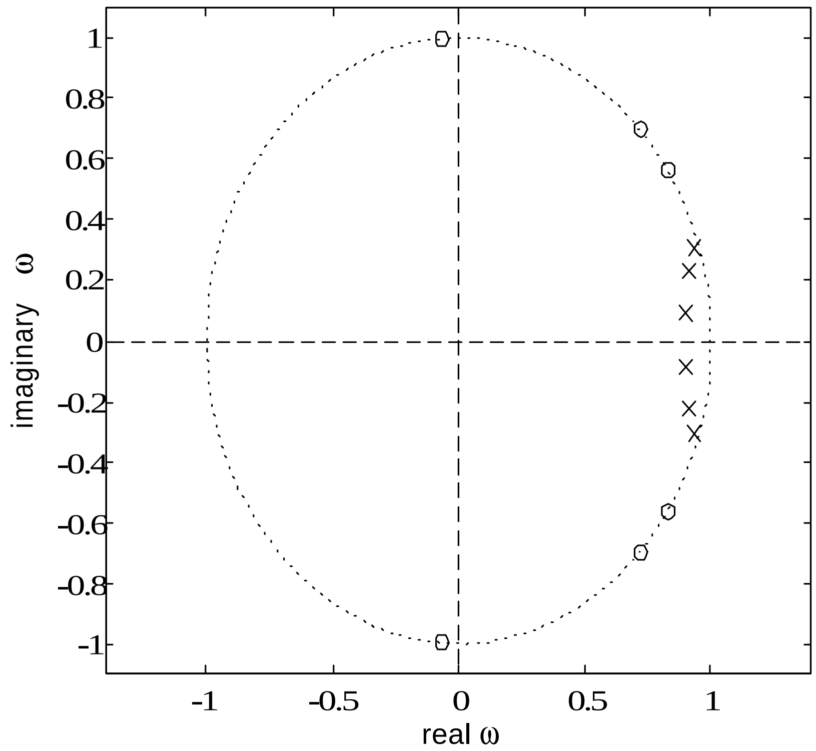




Discrete-Time Filter Response

$$H(z)=\frac{N(z)}{D(z)}=\frac{b_1 z^M + b_2 z^{M-1} + \cdots + b_M z + b_{M+1}}{a_1 z^N + a_2 z^{N-1} + \cdots + a_M z + a_{M+1}}$$

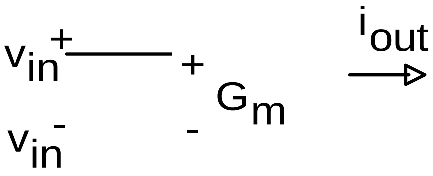




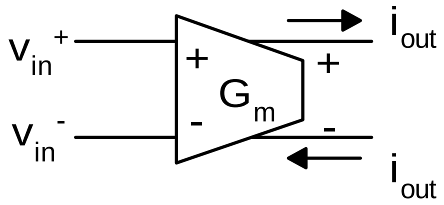
Continuous-Time Filter
(Gm-C Filter)

Gm Amplifier Basics

- **Transconductance (Gm) Amplifier**



Single-Ended

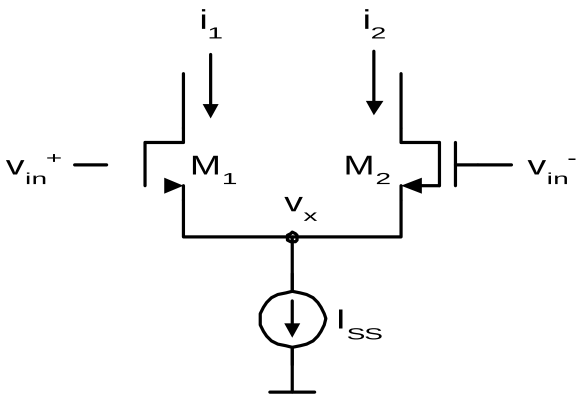


Fully-Differential

- **Transconductance Amplifier Characteristics**
  - ➔ Linearity $i_{out} \equiv G_m \cdot v_{in} = G_m \cdot \left(v_{in}^{+} - v_{in}^{-}\right)$
  - ➔ I/O Impedances $R_{in} = \infty, \quad R_{out} = \infty$
  - ➔ Operating (Dynamic) Range
  - ➔ Frequency Characteristics
  - ➔ Electrically Programmable Gm Value

Gm Amp Circuit Examples

● Simple Differential Pair Example

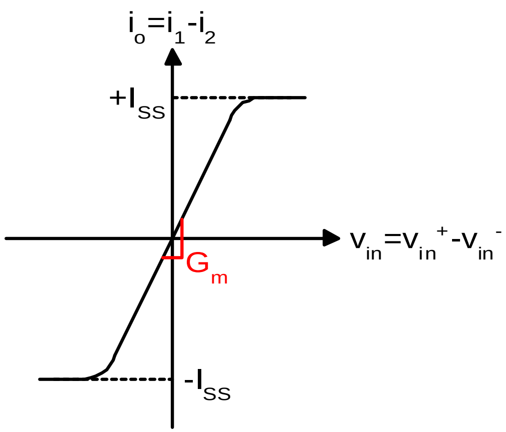


$$i_1 = \frac{\beta}{2}\left(v_{in}^{+} - v_x - V_{TH}\right)^2$$

$$i_2 = \frac{\beta}{2}\left(v_{in}^{-} - v_x - V_{TH}\right)^2$$

$$\rightarrow i_1 + i_2 = I_{SS}$$

$$i_{out} \equiv i_1 - i_2 = \frac{\beta}{2} v_{in} \sqrt{\frac{2I_{SS}}{\beta/2} - v_{in}^2}$$

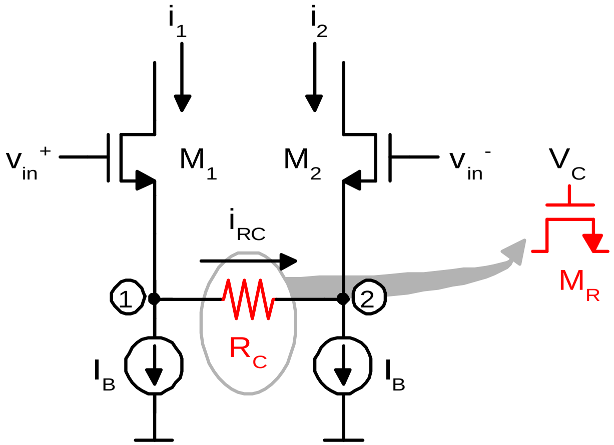


for large I_{SS}, small β & v_{in}

$$i_{out} \approx \sqrt{\beta I_{SS}} \cdot v_{in}$$

$$\therefore G_m \cong \sqrt{\beta I_{SS}}$$

● Differential Pair w/ Degenerate Resistor Example



$$i_1 = i_{R_C} + I_B$$
$$i_2 + i_{R_C} = I_B$$
$$\rightarrow i_1 - i_2 = 2 \cdot i_{R_C}$$

$$v_{in}^+ - v_{gs1} - i_{R_C} R_C + v_{gs2} = v_{in}^-$$

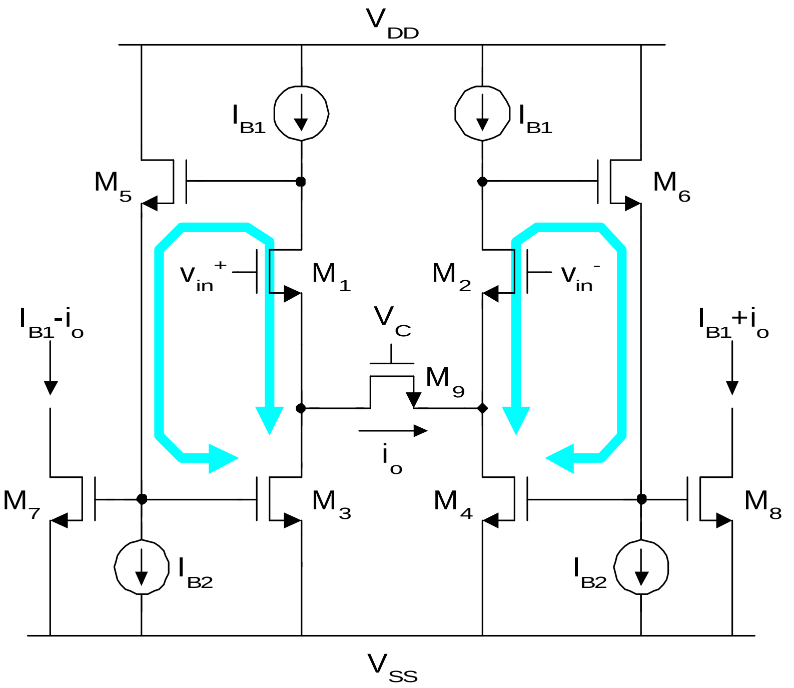
$$\therefore i_{out} \equiv i_1 - i_2 = \frac{2}{R_C}\left\{\Delta v_{in} - \left(v_{gs1} - v_{gs2}\right)\right\}$$

for large β and small $\sqrt{i_1} - \sqrt{i_2}$

$$v_{gs1} - v_{gs2} = \left(V_{TH} + \sqrt{2 i_1/\beta}\right) - \left(V_{TH} + \sqrt{2 i_2/\beta}\right) = \sqrt{2/\beta}\left(\sqrt{i_1} - \sqrt{i_2}\right) \approx 0$$

$$\therefore G_m \equiv \frac{\partial i_{out}}{\partial \Delta v_{in}} = \frac{i_{out}}{\Delta v_{in}} \cong \frac{2}{R_C}$$

● Differential Pair w/ Degenerate Resistor & Feedback Example



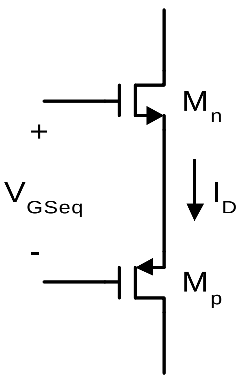
Negative feedback loops (M_1-M_5-M_3 & M_2-M_6-M_4) forces constant currents I_{B1} to flow through M_1 and M_2 ➔ $v_{gs1} = v_{gs2}$

TC Amp using MOS in Saturation

- **Current in Saturation MOSFET** $i_{ds} = \frac{\beta}{2}\left(v_{gs} - V_{TH}\right)^2$

- **Basic Principle** $A^2 - B^2 = (A + B)\cdot(A - B) = k\cdot\left(v_{in}^{+} - v_{in}^{-}\right)$

- **Composite Transistor**

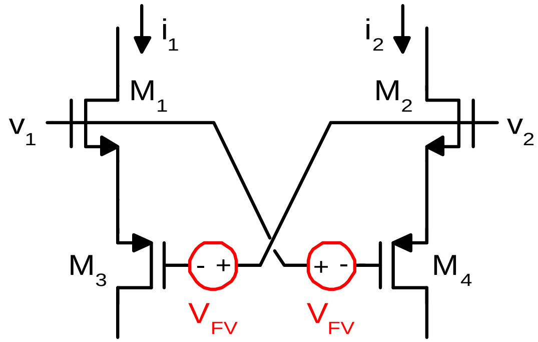


$$I_D = \frac{\beta_{eq}}{2}\left(V_{GSeq} - V_{THeq}\right)^2$$

$$V_{GSeq} \equiv V_{GSN} + V_{SGP}$$

$$V_{THeq} \equiv V_{THN} + \left|V_{THP}\right|$$

$$\beta_{eq} \equiv \frac{1}{2}\frac{\beta_N \beta_P}{\left(\sqrt{\beta_N/2} + \sqrt{\beta_P/2}\right)^2}$$



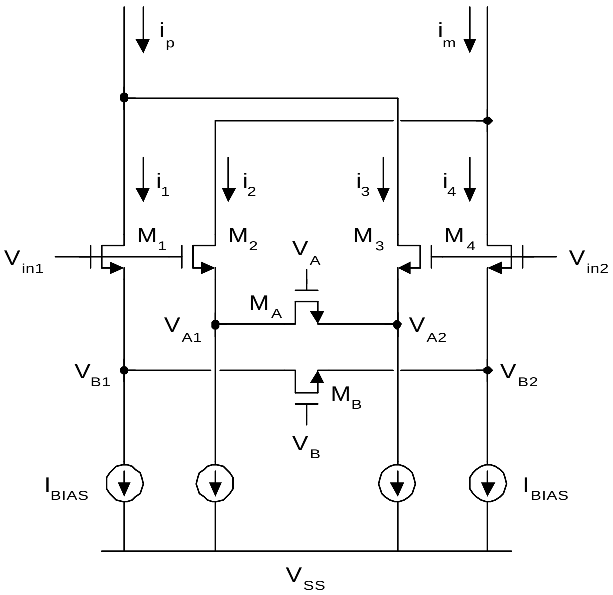
$$i_{out} = 2\beta_{eq}\left[V_{FV} - V_{THeq}\right]\cdot\left(v_1 - v_2\right)$$

TC Amp using MOS in Linear

- **Current in Saturation MOSFET** $$i_{ds} = \beta\left[(v_{gs} - V_{TH})v_{ds} - \frac{1}{2}v_{ds}^2\right]$$

- **Basic Principle** $$(A - C^2) - (B - C^2) = A - B \propto v_{in}^+ - v_{in}^-$$

- **Example**

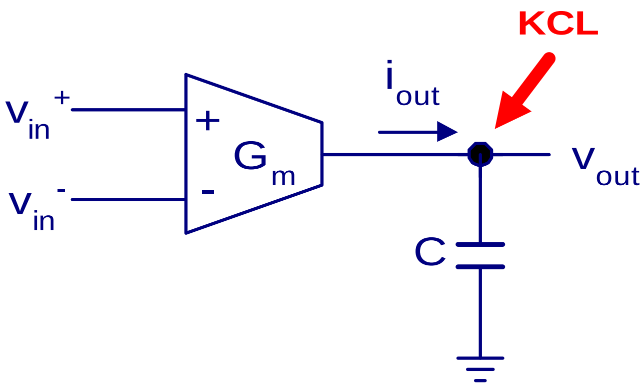


$$i_{out} = i_p - i_m = (i_1 + i_3) - (i_2 + i_4) = (i_1 - i_4) - (i_2 - i_3)$$

$$i_1 - i_4 = 2i_B$$
$$= 2\beta\left[(V_B - V_{B2} - V_{TH})(V_{B1} - V_{B2}) - \frac{1}{2}(V_{B1} - V_{B2})^2\right]$$

$$i_2 - i_3 = 2i_A$$
$$= 2\beta\left[(V_A - V_{A2} - V_{TH})(V_{A1} - V_{A2}) - \frac{1}{2}(V_{A1} - V_{A2})^2\right]$$

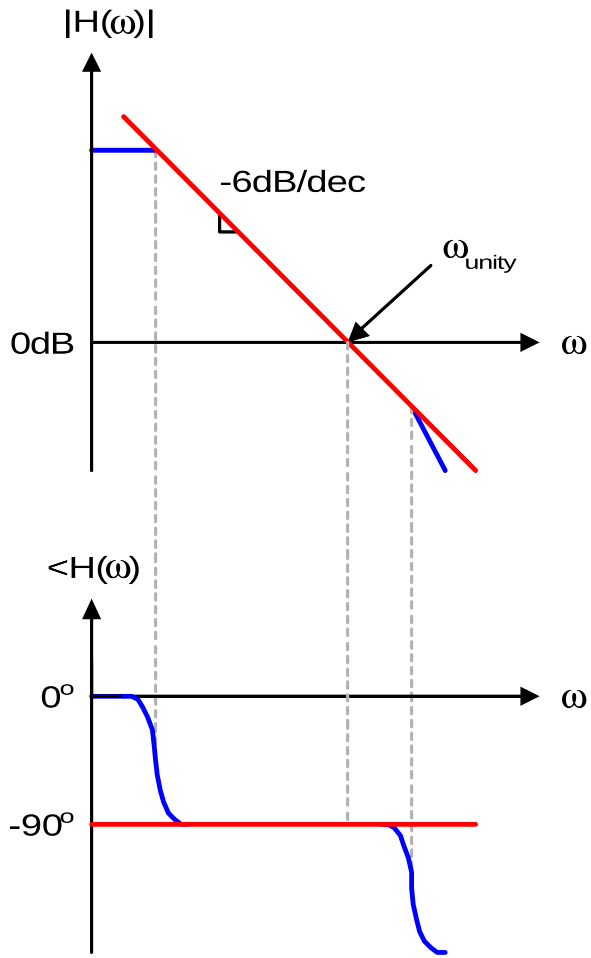
Gm-C Integrator



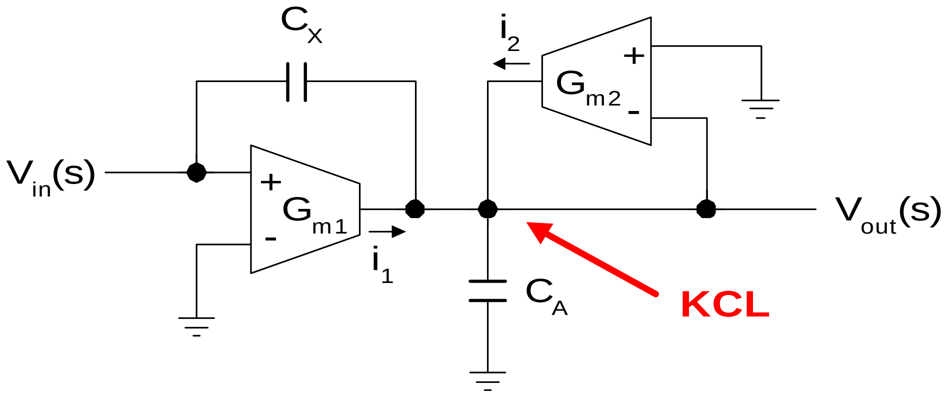
$$H(s) \equiv \frac{v_{out}}{v_{in}} = \frac{G_m}{sC}$$

$$\omega_{unity} = \frac{G_m}{C}$$

critical parameters to determine filter frequency characteristics



1^{st}-Order Gm-C Filter



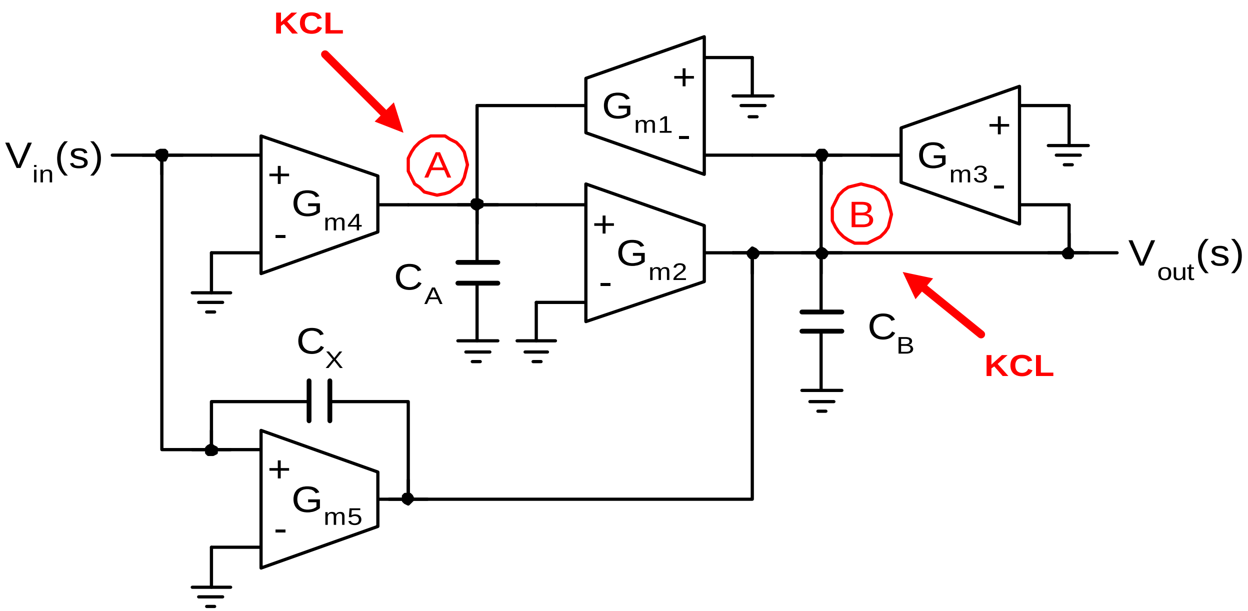
$$H(s) \equiv \frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{C_X}{C_A + C_X}\right)s + \left(\frac{G_{m1}}{C_A + C_X}\right)}{s + \left(\frac{G_{m2}}{C_A + C_X}\right)} \Leftarrow \frac{k_1 s + k_0}{s + \omega_0}$$

$$G_{m1} = k_0 \cdot (C_A + C_X)$$

$$G_{m2} = \omega_0 \cdot (C_A + C_X)$$

$$C_X = \frac{k_1}{1 - k_1} C_A$$

2^{nd}-Order Gm-C Filter (Biquad)



$$H(s) \equiv \frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{C_X}{C_B + C_X}\right)s^2 + \left(\frac{G_{m5}}{C_B + C_X}\right)s + \frac{G_{m2}G_{m4}}{C_A(C_B + C_X)}}{s^2 + \left(\frac{G_{m3}}{C_B + C_X}\right)s + \frac{G_{m1}G_{m2}}{C_A(C_B + C_X)}} \Leftarrow \frac{k_2 s^2 + k_1 s + k_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

Characteristic Function

$$\text{denominator} = s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2$$

$$\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_A(C_B + C_X)}}$$

$$Q = \sqrt{\frac{G_{m1}G_{m2}}{G_{m3}^2}\frac{C_B + C_X}{C_A}}$$

Filtering Operations

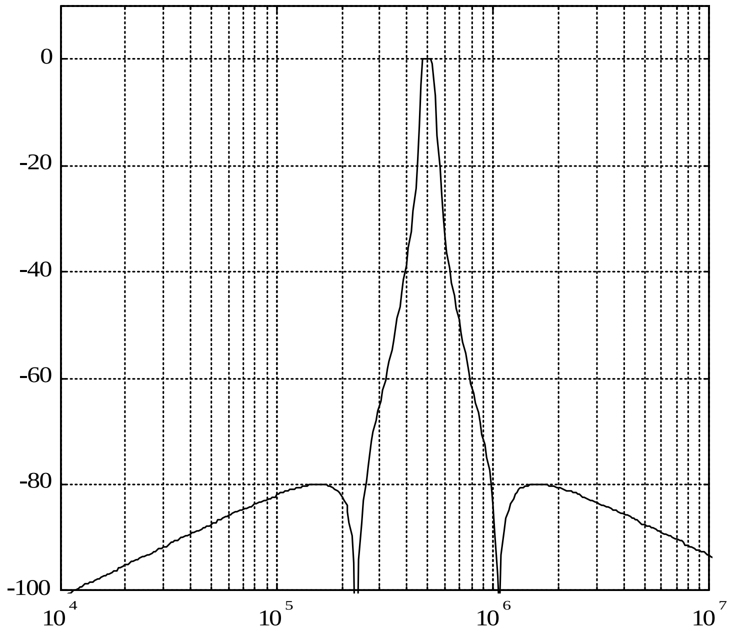
$$k_2 = \frac{C_X}{C_B + C_X} \qquad k_1 = \frac{G_{m5}}{C_B + C_X} \qquad k_0 = \frac{G_{m2}G_{m4}}{C_A(C_B + C_X)}$$

- ✓ Lowpass $k_2 = k_1 = 0$
- ✓ Highpass $k_1 = k_0 = 0$
- ✓ Bandpass $k_2 = k_0 = 0$

High-Order Gm-C Filter w/ Biquads

6th-order Elliptic BPF

- ✓ Passband : 480kHz~520kHz
- ✓ Passband Ripple : < 0.05dB
- ✓ Stopband Attenuation : > 80dB



- $H_1(s)$

$G_{m1}=G_{m2}=17.06\mu$, $G_{m3}=0.80\mu$

$G_{m4}=4.18\mu$, $G_{m5}=0\mu$

$C_A=C_B=5p$, $C_X=0.319p$

- $H_2(s)$

$G_{m1}=G_{m2}=16.19\mu$, $G_{m3}=1.52\mu$

$G_{m4}=0.21\mu$, $G_{m5}=0\mu$

$C_A=C_B=5p$, $C_X=0.319p$

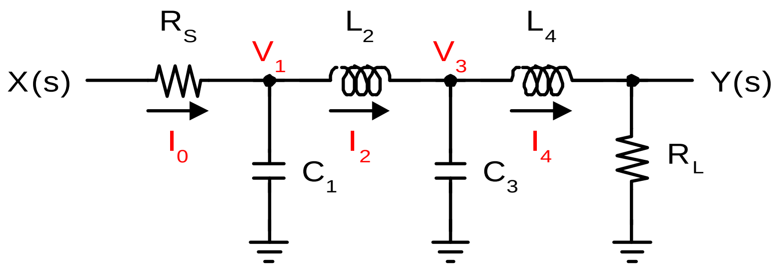
- $H_3(s)$

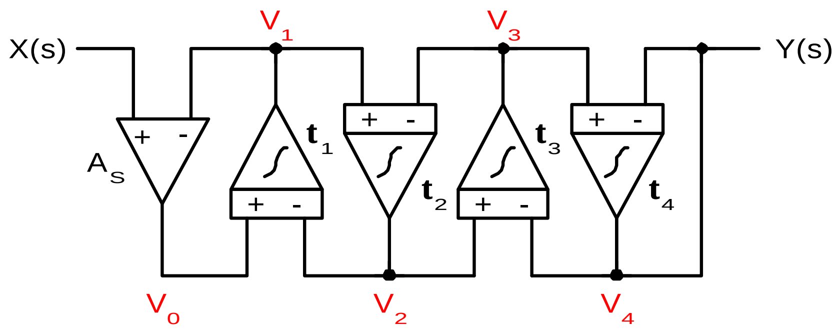
$G_{m1}=G_{m2}=14.89\mu$, $G_{m3}=0.67\mu$

$G_{m4}=0\mu$, $G_{m5}=1.89\mu$

$C_A=C_B=5p$, $C_X=0p$

High-Order Gm-C Filter from RLC





$$I_0 = (X - V_1)/R_S$$
$$V_1 = (I_0 - I_2)/sC_1$$
$$I_2 = (V_1 - V_3)/sL_2$$
$$V_3 = (I_2 - I_4)/sC_3$$
$$I_4 = (V_3 - I_4 R_L)/sL_2$$

$$V_0 = A_S (X - V_1)$$
$$V_1 = (V_0 - V_2)/s\tau_1$$
$$V_2 = (V_1 - V_3)/s\tau_2$$
$$V_3 = (V_2 - V_4)/s\tau_3$$
$$V_4 = (V_3 - Y)/s\tau_4$$

Frequency Transformation & Impedance Transformation Required

Non-Idealities

- **Finite Output Resistance** $H(s) = \frac{G_m R_{out}}{1 + sR_{out} C}$
  - ✓ Finite DC Gain : BPF, HPF
  - ✓ Reduced Phase (>-90°): Q decreases
  - ✓ Cascode Output Stage & Compensation Technique
- **Finite Parasitic Pole Frequency** $H(s) = \frac{G_m R_{out}}{1 + sR_{out} C} \cdot \frac{1}{1 + s/p_{internal}}$
  - ✓ Increased Phase (<-90°): Q increases
  - ✓ Careful Design
- **Input Capacitance Loading** $H(s) = \frac{G_m}{s(C + C_{input})}$
  - ✓ Frequency Change
  - ✓ Dummy Cells

Linearity Problems

- **Nonlinearity** $I_{out}(t) \equiv a_1 V_{in}(t) + a_2 [V_{in}(t)]^2 + a_3 [V_{in}(t)]^3 + \cdots$

- **Harmonic Distortion** $V_{in}(t) = A \sin(\omega_0 t)$

$$I_{out}(t) = I_1 \sin(\omega_0 t) + I_2 \sin(2\omega_0 t) + I_3 \sin(3\omega_0 t) + \cdots$$

$$THD\ [dB] \equiv 10 \log\left(\frac{I_2{}^2 + I_3{}^2 + I_4{}^2 + \cdots}{I_1{}^2}\right)$$

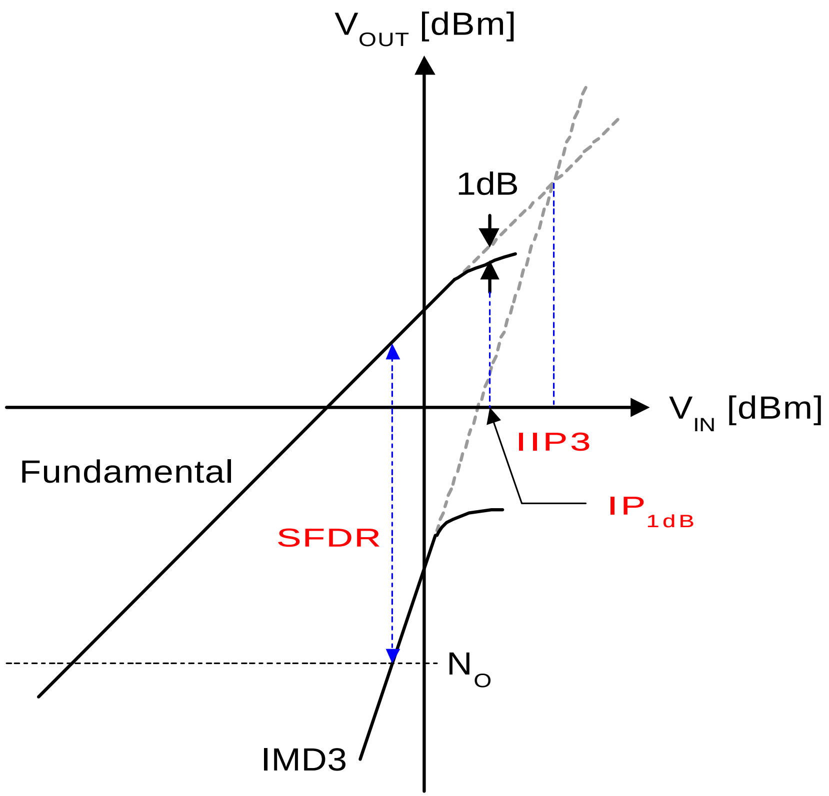
- **Inter Modulation Distortion** $V_{in}(t) = A[\sin(\omega_1 t) + \sin(\omega_2 t)],\ \omega_1 \approx \omega_2$

  ✓ IMD2 → $\omega_1 + \omega_2,\ \omega_1 - \omega_2$

  ✓ IMD3 → $\omega_1 + 2\omega_2,\ 2\omega_1 + \omega_2,\ 2\omega_1 - \omega_2,\ \omega_1 - 2\omega_2,$

Dynamic Range Performance

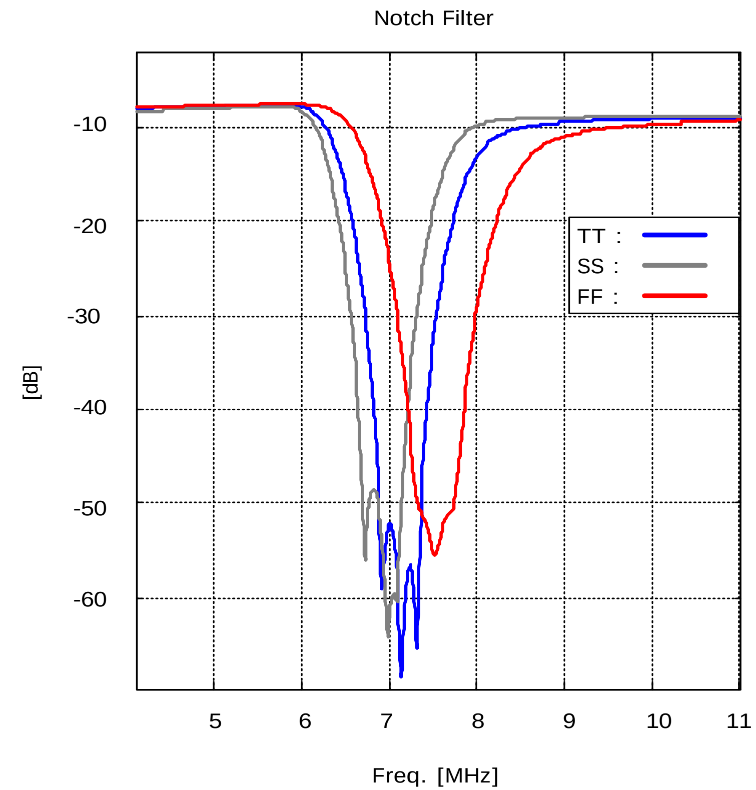
- IIP3 : Input-Referred Third-Order Intercept Point
- IP_{1dB} : Input-Referred 1-dB Compression Point
- SFDR : Spurious-Free Dynamic Range



Gm Variations

- Values of G_m might change due to

  process variation, temperature, aging, ….

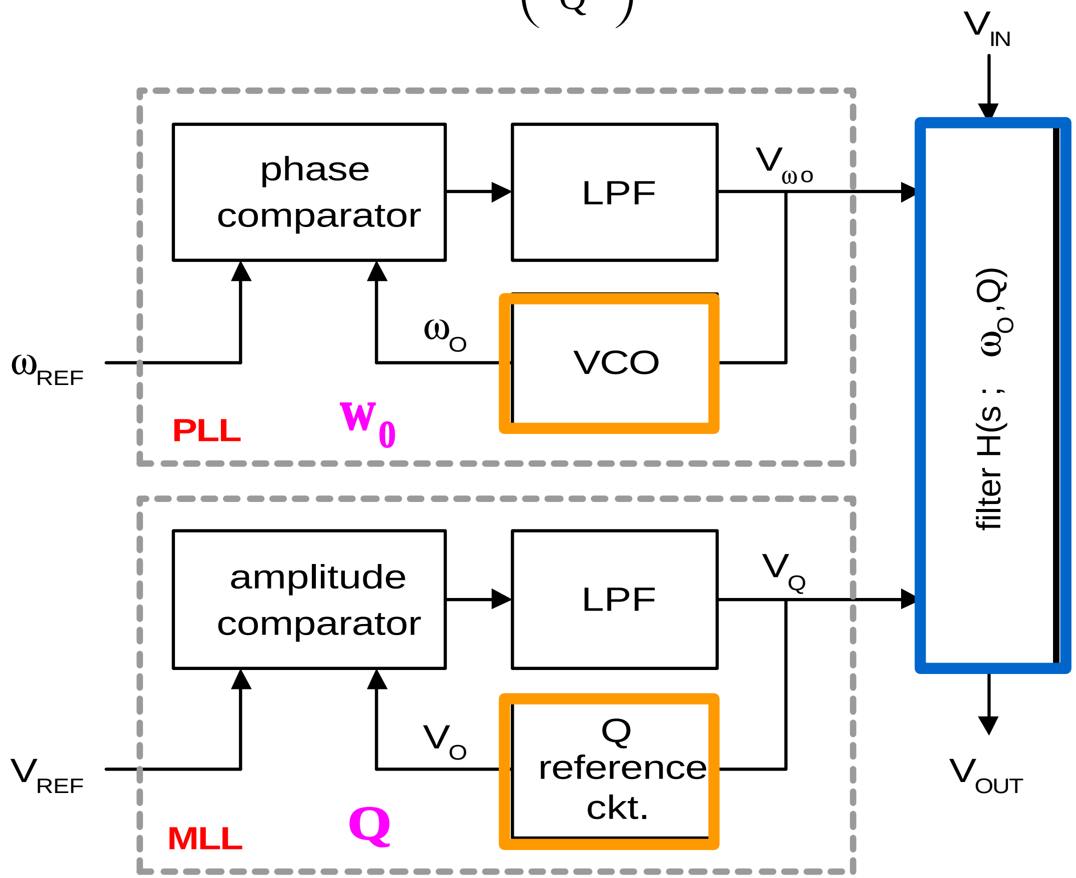


- freq. change
- Q change

- **Trimming** after chip fabrication possible, but one-time event.

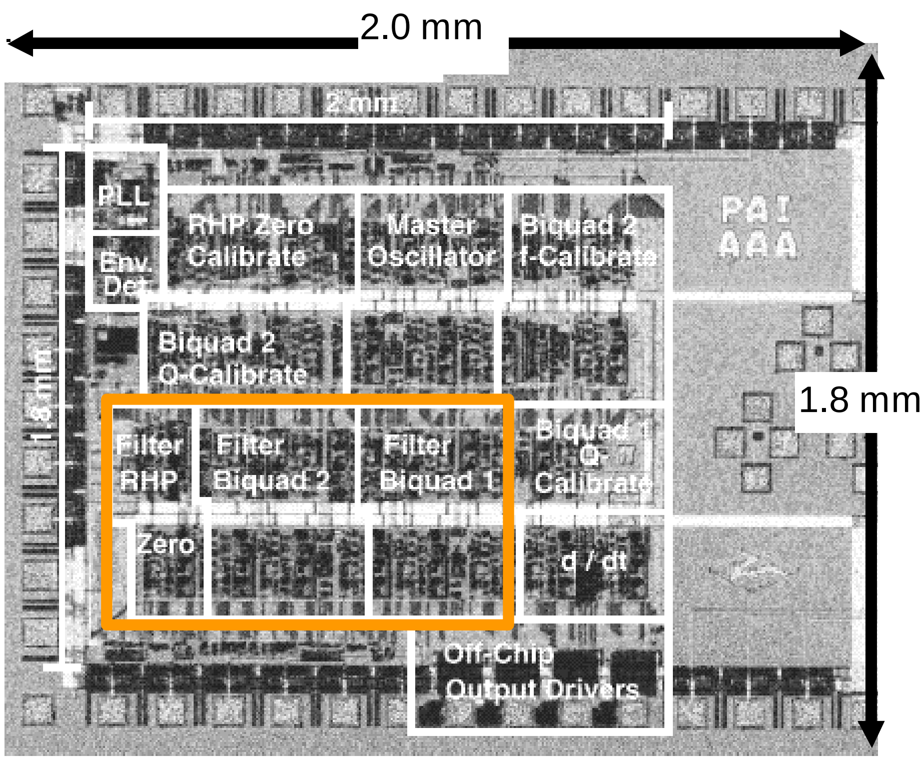
On-Chip Tuning Example for Biquad

$$H(s) = \frac{D(s)}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$



Recent Advance Example 1

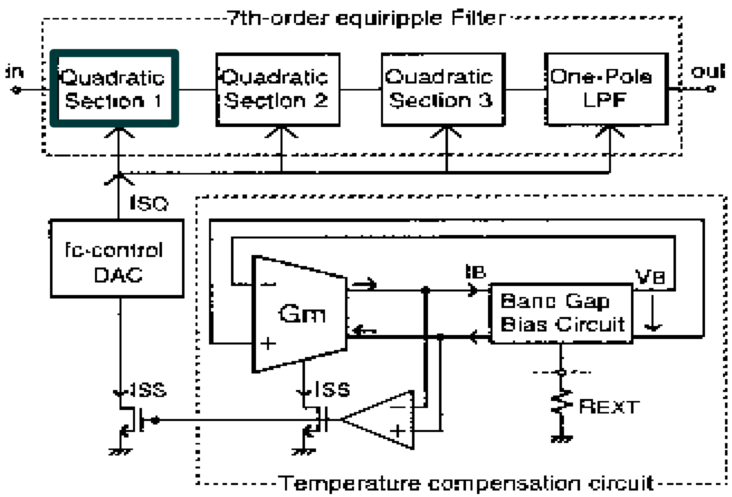
55 Mbps Equalizer for Magnetic Storage Read Channels

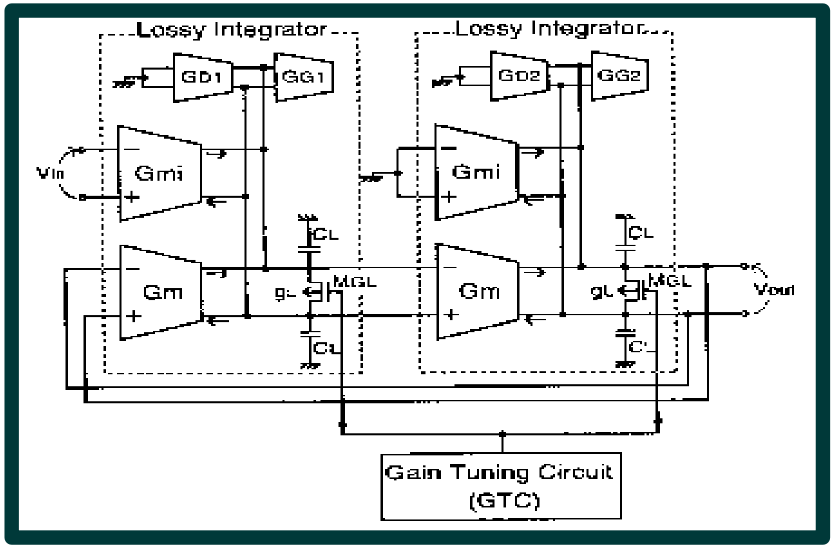


- 4-pole & 1-zero
- Equalization
- for Peak-Detect Scheme
- 2.0-µm CMOS Technology
- SNR > 57 dB, THD < 1%
- Power = 40 mW
- UCLA (📖 *IEEE JSSC*, 1994)

Recent Advance Example 2

7-th Order Equiripple Gm-C Filter

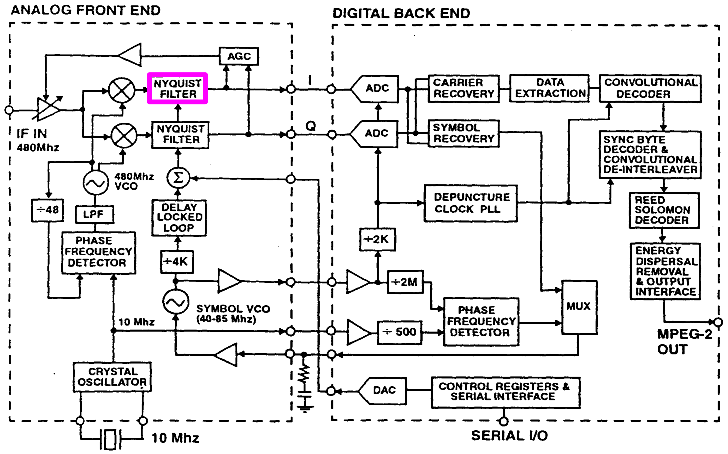


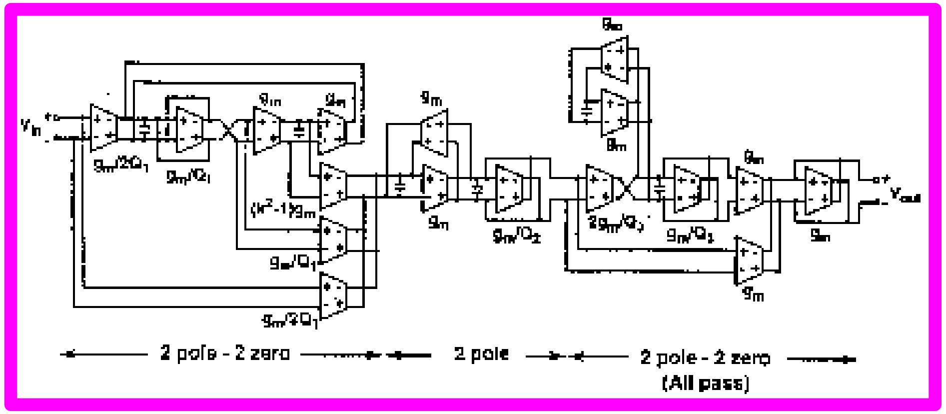


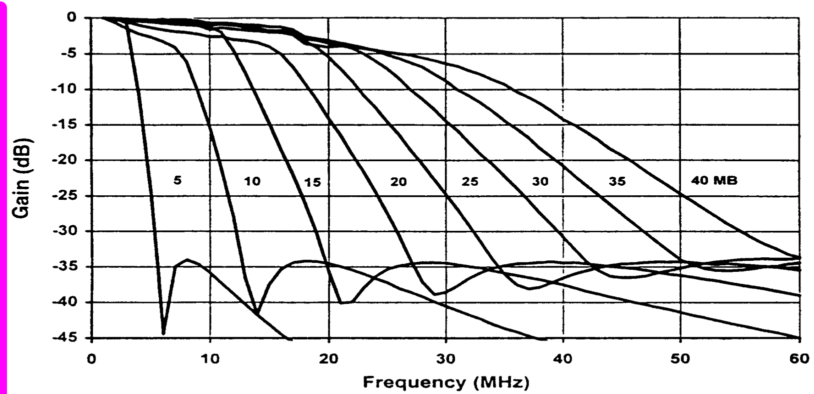
- 0.4-µm CMOS Technology
- f_{cutoff} = 50 MHz
- THD < 1 %, Power = 100 mW (3 V)
- Hitachi (📖 *IEEE ISSCC*, 1997)

Recent Advance Example 3

Raised-Cosine Matched Filter for 480 MHz QPSK Demodulator for DBS



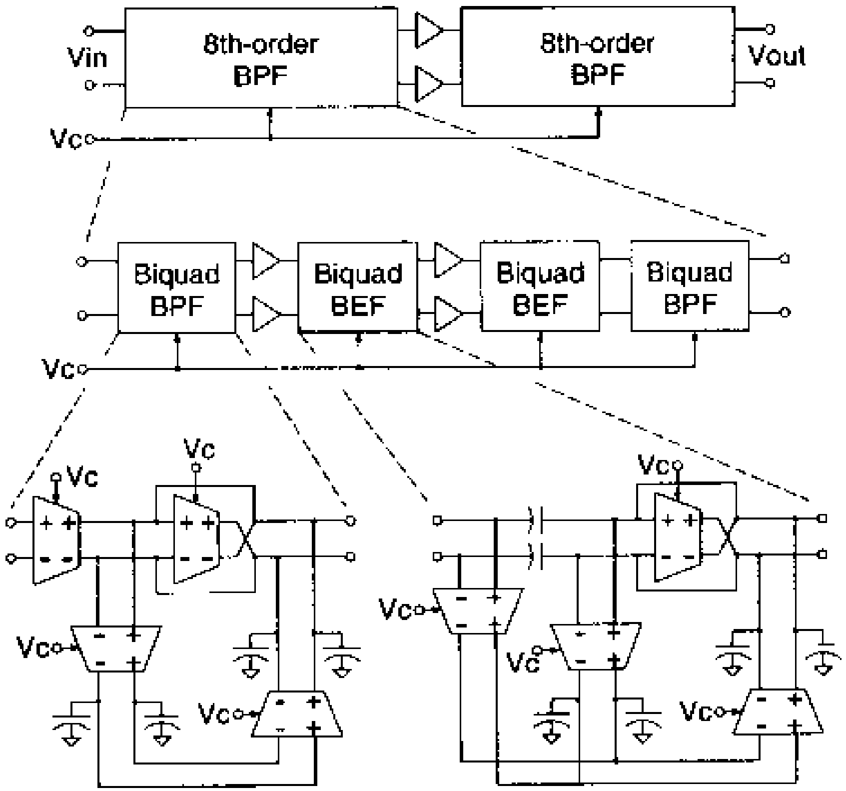




Analog Devices (📖 *IEEE ISSCC*, 1997)

Recent Advance Example 4

16th-order BPF for J-PDC

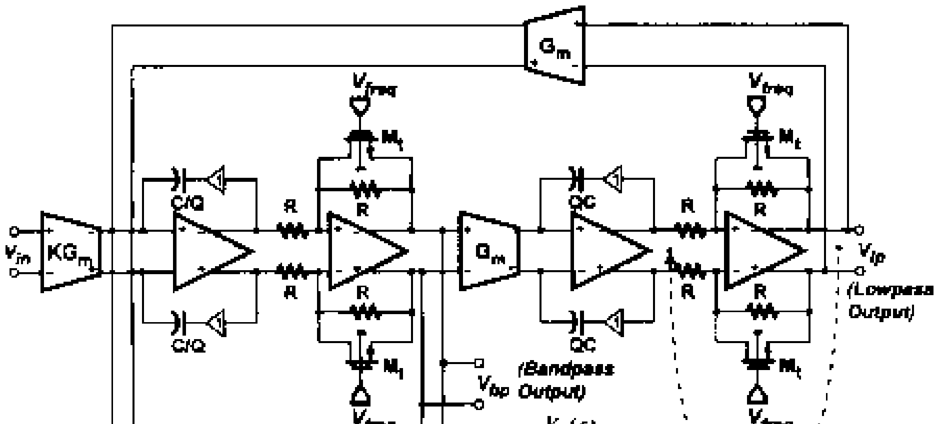


- 2 x 8th-order BPF
- Average of f_o : 450.5kHz
- σ of f_o : $\pm$1.5kHz
- Tuning Range of f_o : 240~770kHz
- 0.35-μm CMOS
- Power : 4.8mA @ 2.5V
- Active Area : 2.5mm^2
- Fujitsu

  (📖 *IEEE ISSCC*, 1999)

Recent Advance Example 5

7th-order Equiripple Group Delay LPF



$$H_{lp}(s) = \frac{V_{lp}(s)}{V_{in}(s)} = K\frac{1}{\left(\frac{s}{w_c}\right)^2 + \left(\frac{s}{w_0 Q}\right) - 1} \quad \text{where } w_0 = \frac{\alpha G_m}{C}$$

- 1 + 3 x Biquads
- f_{3dB} : 450.5kHz
- f_{3dB} Accuracy: ±10%
- Group Delay Acc. : ±5%
- 0.25-µm CMOS
- Power Supply : 2.5V
- Chip Area : 3mm^2
- Texas Instruments

(📖 *IEEE ISSCC*, 1999)